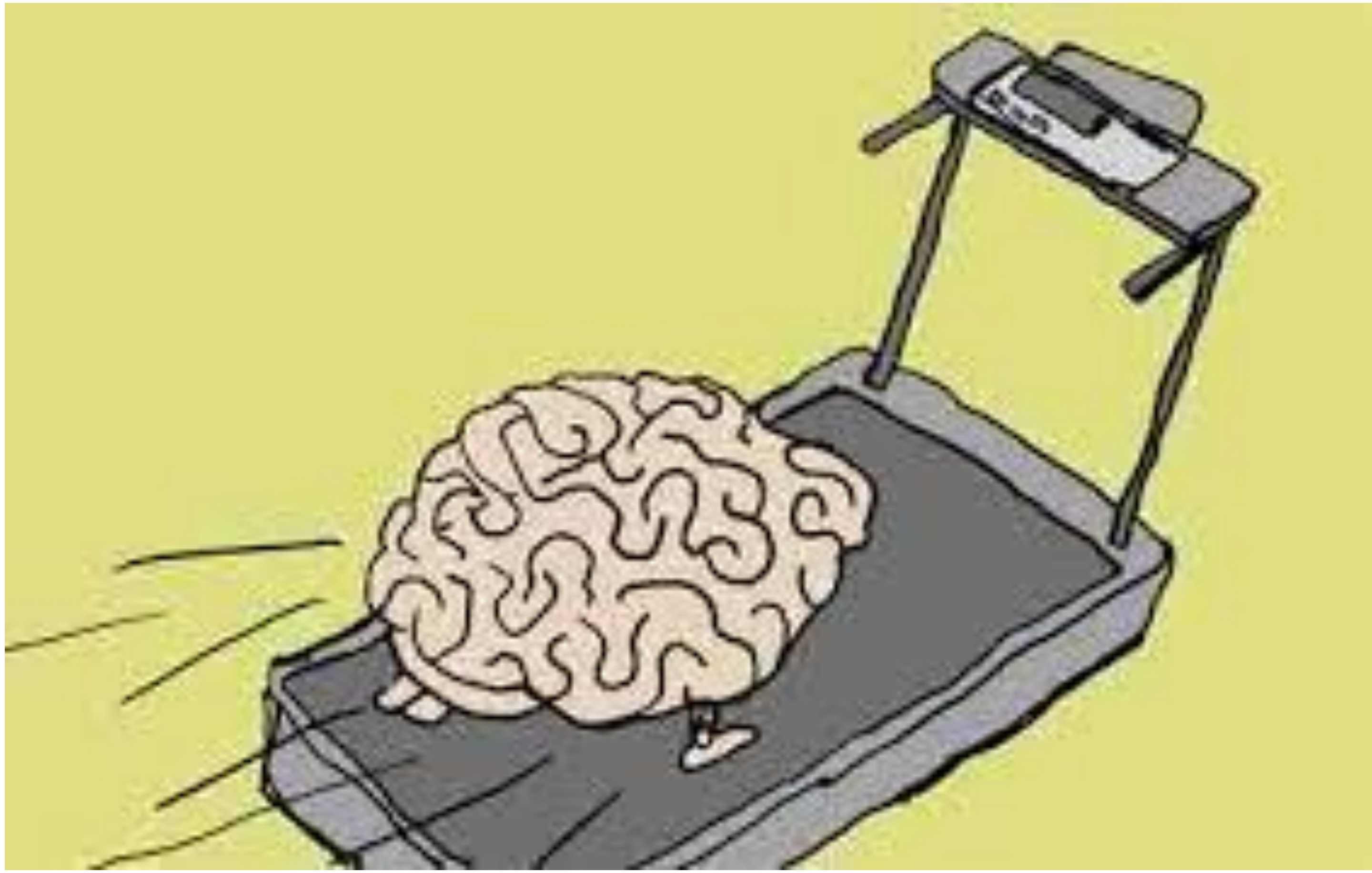
PRODUCTIVE STRUGGLE:

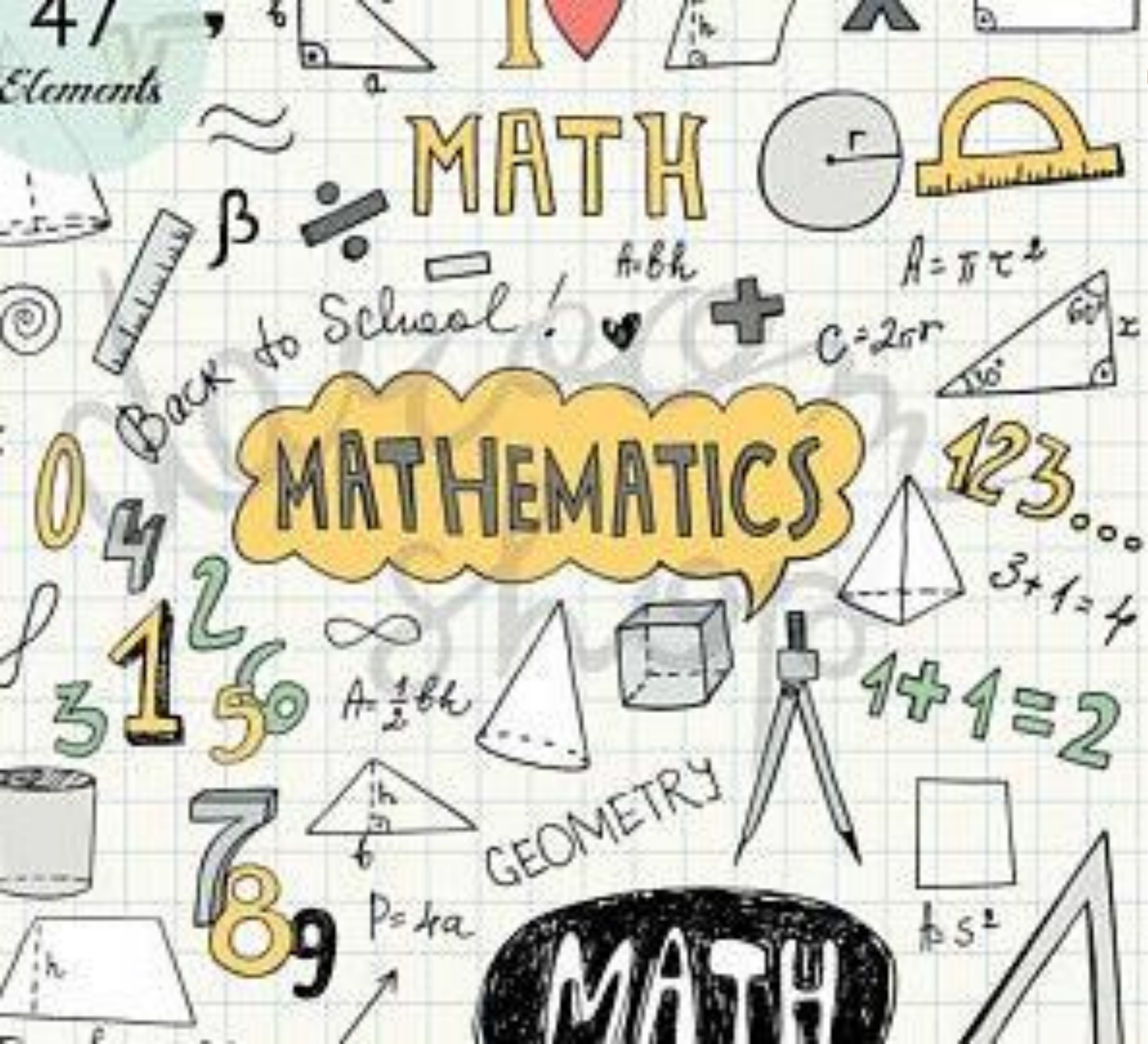
How educators can help students with LDs build resilience and achieve deeper math learning

@DrMaryReid

PRODUCTIVE STRUGGLE:

- to put forth a concerted effort to understand and solve math problems that are not immediately obvious (Hiebert & Grouws, 2007).





PRODUCTIVE STRUGGLE VS VOLUME OF WORK

- Students are most often stressed by volume of tasks
- Repetitious busy work that does not challenge students' thinking does nothing to deepen learning

Dr. Mary Reid, Aug, 2020

THE URGE TO REDUCE STRUGGLE QUICKLY

- Often viewed in a negative light
- Described as a learning challenge
- Teachers attempt to remove the struggle

- Must see students' struggles as meaningful learning opportunities

(Heibert & Wearne, 2003)

PRODUCTIVE STRUGGLE AND STUDENTS WITH LDs

- working memory and processing speed (Fuchs & Fuchs, 2002),
- distinguishing relevant information which impacts reasoning (Maccini & Ruhl, 2001)
- comprehension demands of word problems (Lerner, 2000)
- self-monitoring and self-regulation during problem-solving (Gagnon & Maccini, 2001)
- self-esteem and/or self-efficacy (Maccini & Gagnon, 2000).
- math anxiety (Reid & Reid, 2020)

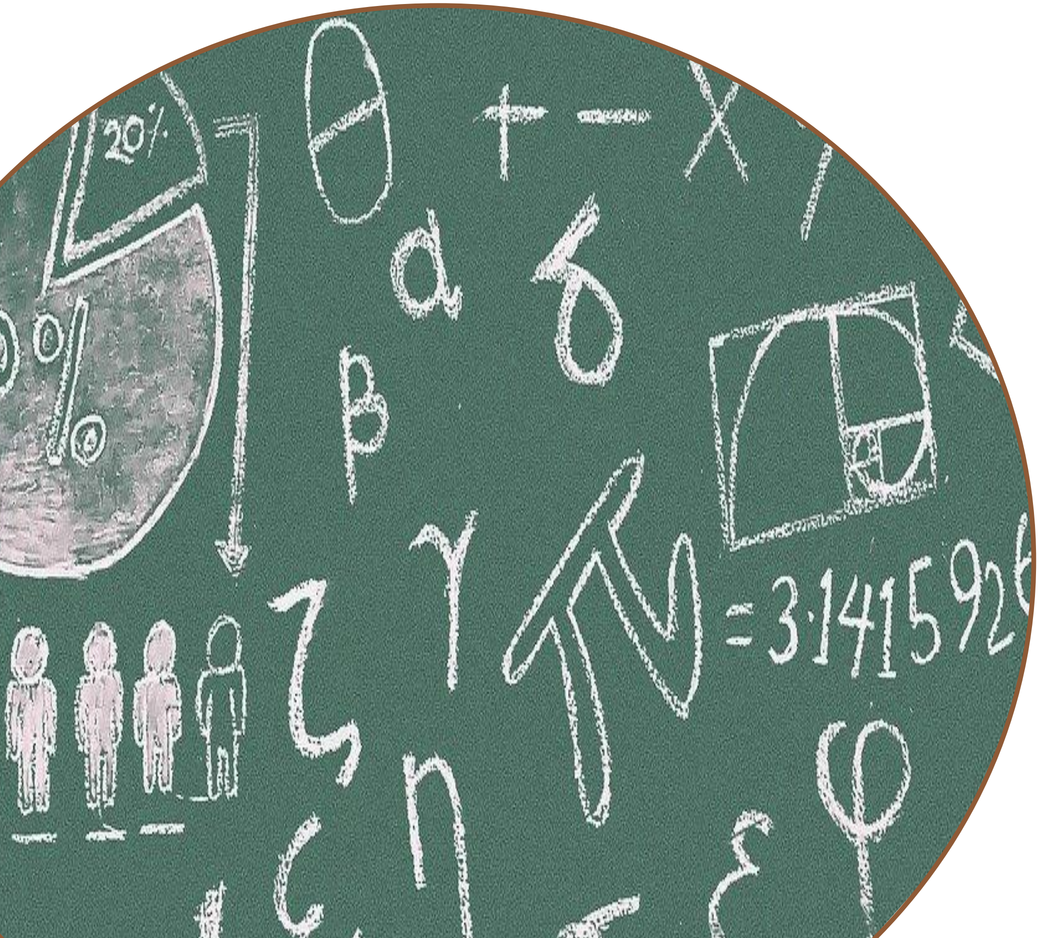
THE URGE TO REDUCE STRUGGLE QUICKLY

- Offer answer
- Make task into simpler procedural tasks
- Direct students' thinking toward an answer, without building connections

Lowers or removes cognitive demand

Math is **NOT** about how many answers you know…
it's how you behave when you don't know.

PRODUCTIVE STRUGGLE AND STUDENTS WITH LDs



When teaching math to students who have challenges, especially those with special education needs, the struggle is often removed (Lynch, Hunt, & Lewis, 2018).

STRUGGLE IS ESSENTIAL

- "Perplexity, confusion or doubt" (Dewey, 1933, p. 12)
- Restructuring disequilibrium (Piaget, 1960)
- Cognitive dissonance (Festinger, 1957)
- Cognitive incongruity (Hatano, 1988)
- “Desirable difficulties” (Bjork & Bjork, 2011, p. 57)

(Jackson & Lambert, 2010)

PRODUCTIVE

- advances thinking
- increases cognitive demand
- enhances intellectual efforts
- deepens understanding at a conceptual level

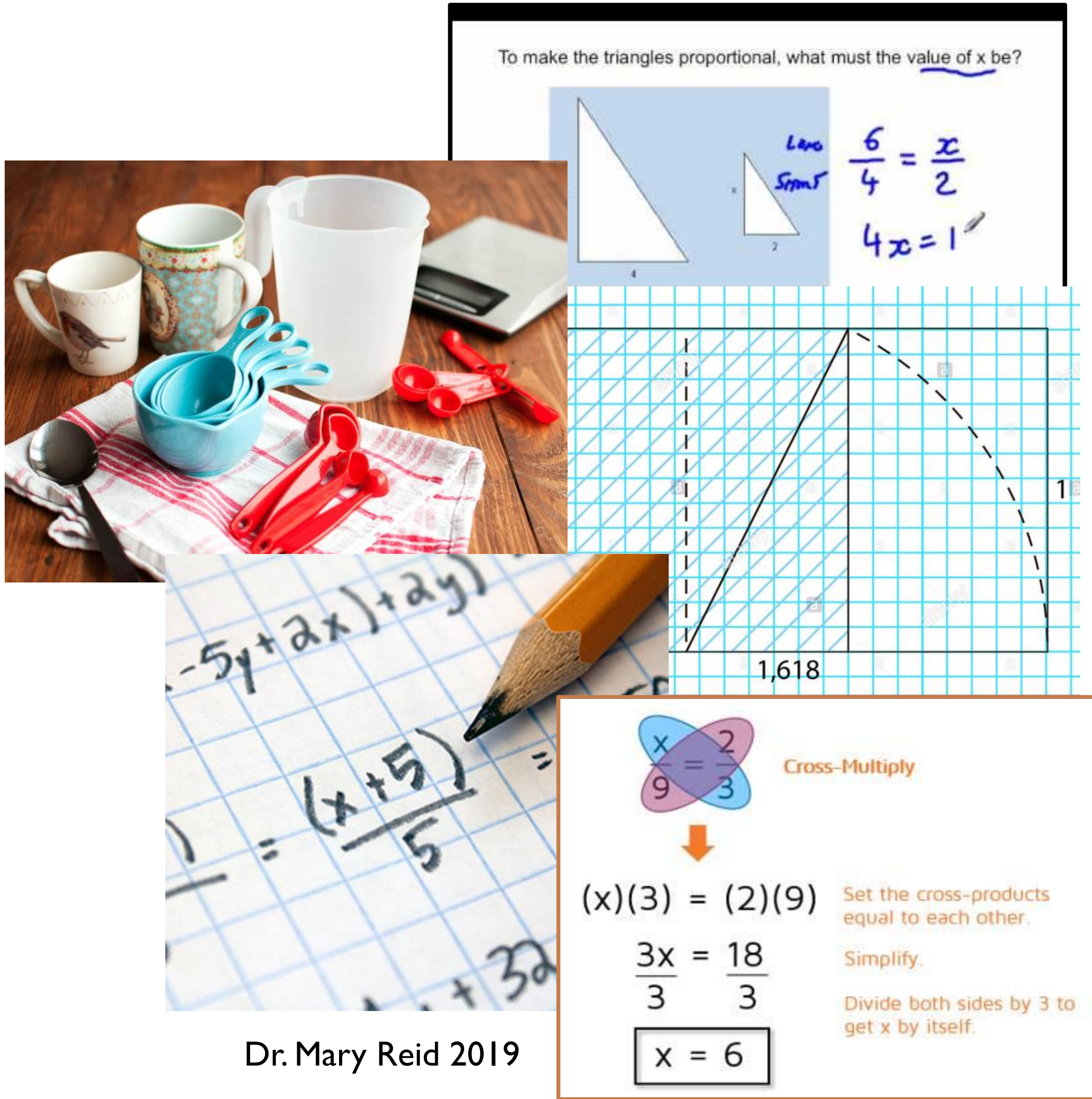
DESTRUCTIVE

- struggle feels permanent
- cycle of failure
- personal, the flaws are within
- fixed mindset

HOW TEACHERS RESPOND TO STUDENTS' STRUGGLE

Study conducted by Warshauer, (2015)

- 327 grade 6 & 7 students
- six teachers
- three middle schools in Texas, diverse communities
- video recorded each teacher - 6 to 8 math lessons on proportional reasoning
- total of 52.5 observation hours
- captured struggle interactions
- coding of tasks, teachers` responses, and resolution of student struggles



KINDS OF STUDENT STRUGGLES
186 OBSERVED SITUATIONS

Kind of struggles	Frequency (%)
1. Getting started	24

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4. Express misconception and errors	13

$0.127 > 0.3$?

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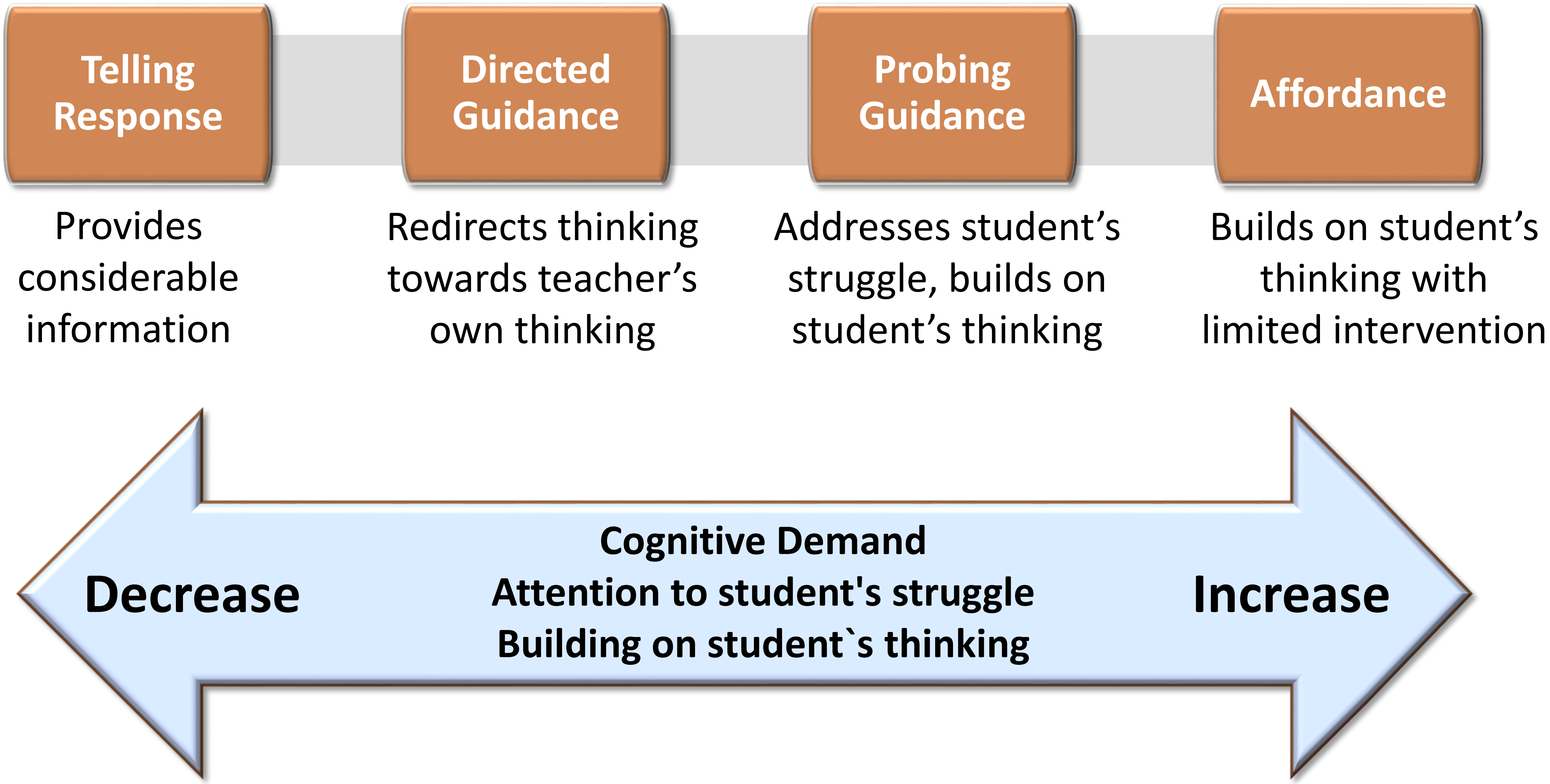
How teachers respond to students' struggle…



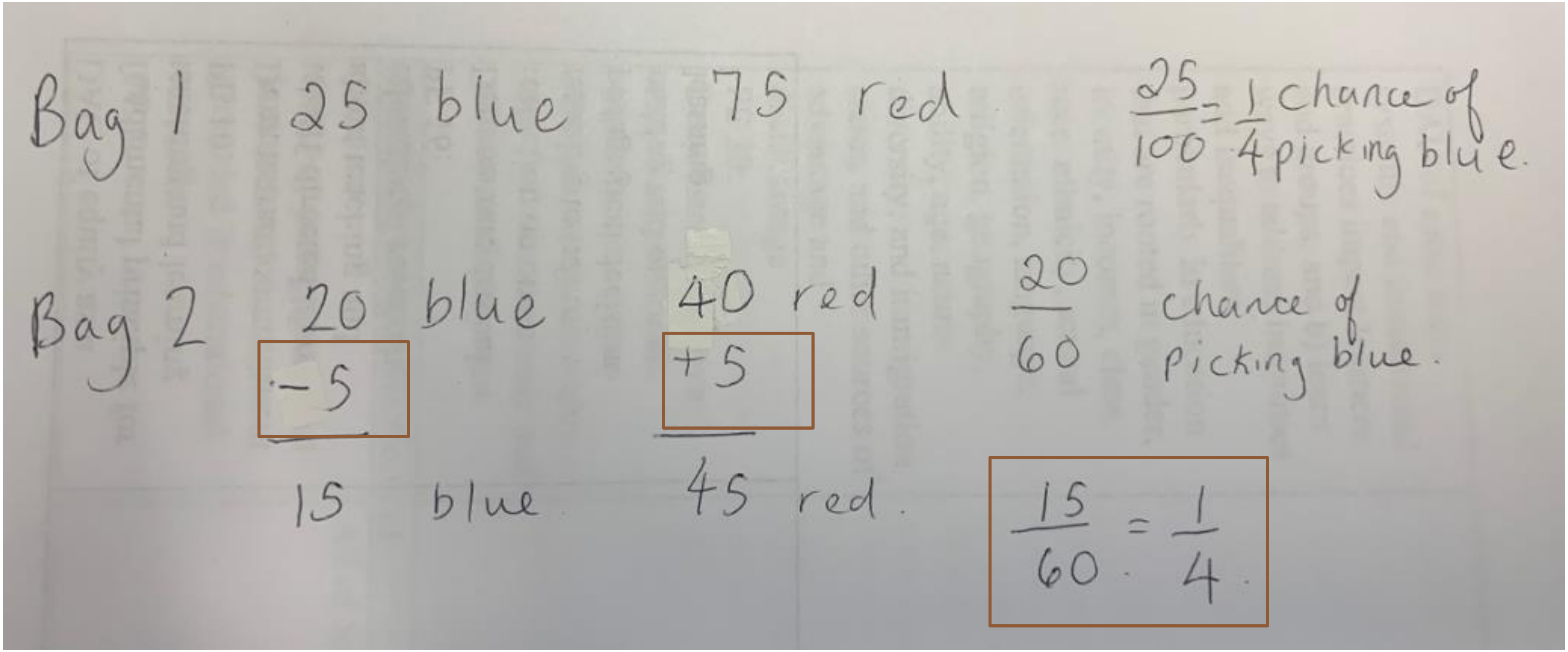
(Warshauer, 2015)

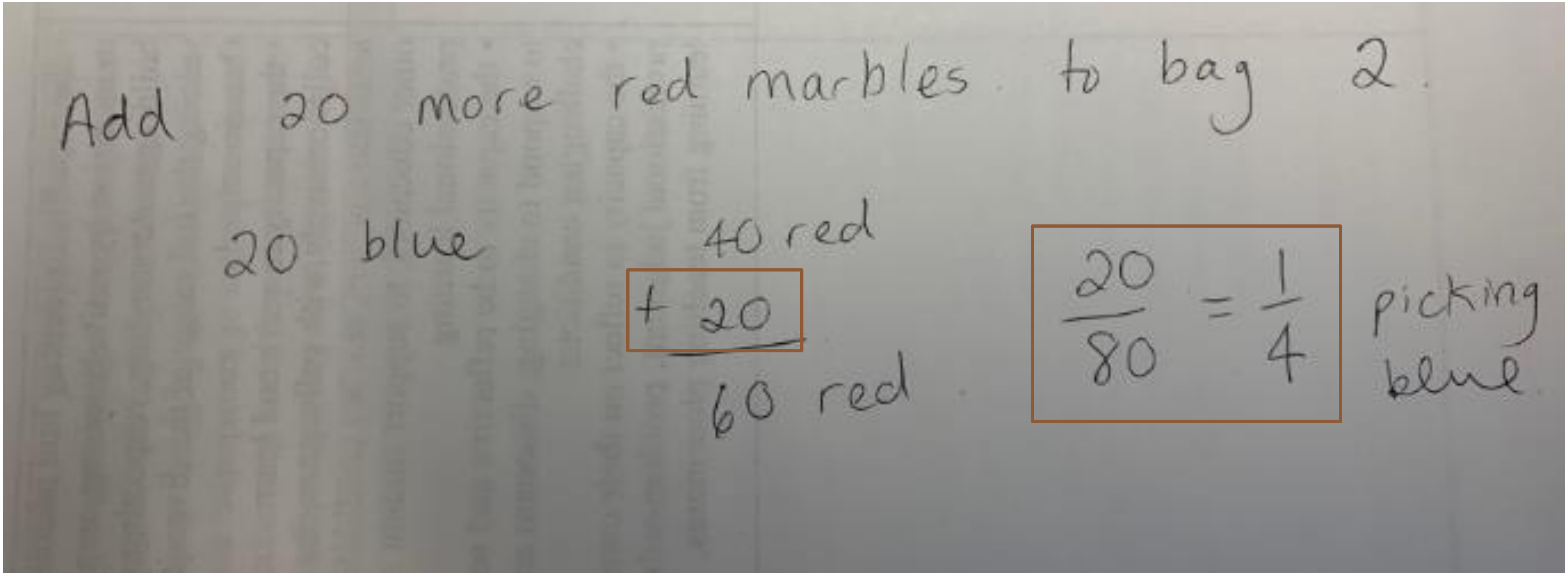
TASK

- Bag 1 has a total of 100 marbles of which 75 are red and 25 are blue.
- Bag 2 has a total of 60 marbles of which 40 are red and 20 are blue.

How can you change bag 2 to have the same chance of getting a blue marble as bag 1?

Explain your answer.


Bag 1 25 blue 75 red 25/100 = 1/4 chance of picking blue.
Bag 2 20 blue 40 red 20/60 chance of picking blue.
-5
+5
15 blue 45 red.
15/60 = 1/4.

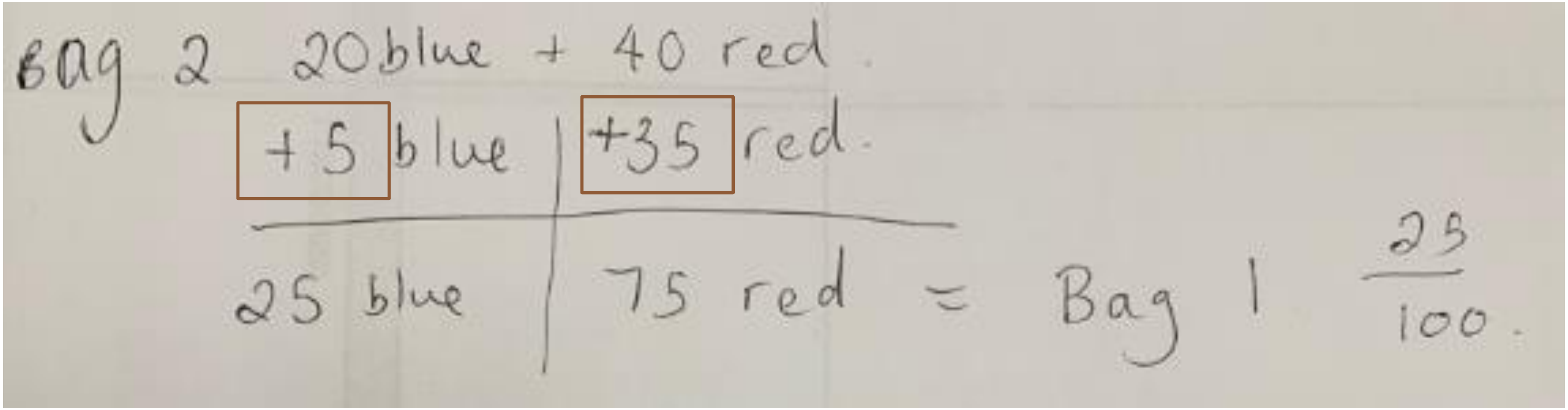

Add 20 more red marbles to bag 2.
20 blue
40 red
+ 20
60 red.
20/80 = 1/4
picking blue

Bag 2

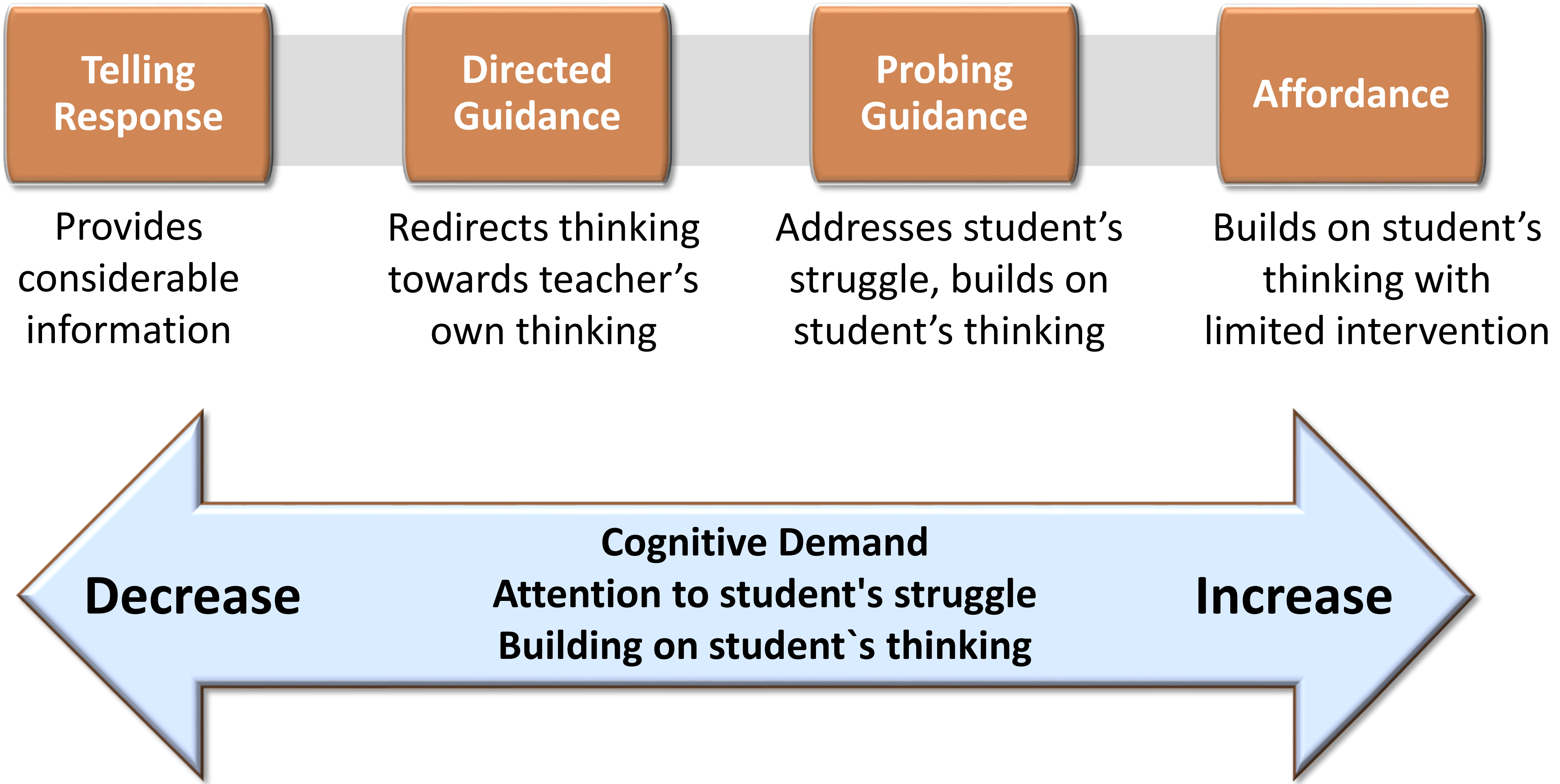
20 blues → −10 blues = 10 blue

40 reds → − 10 reds = 30 red.

$\frac{10}{40} = \frac{1}{4}$ chance of picking Blue


Bag 2 20 blue + 40 red.
+ 5 blue
+35 red.
25 blue
75 red = Bag 1
25/100.

How teachers respond to students' struggle…



(Warshauer, 2015)

TASK

- Bag 1 has a total of 100 marbles of which 75 are red and 25 are blue.
- Bag 2 has a total of 60 marbles of which 40 are red and 20 are blue.
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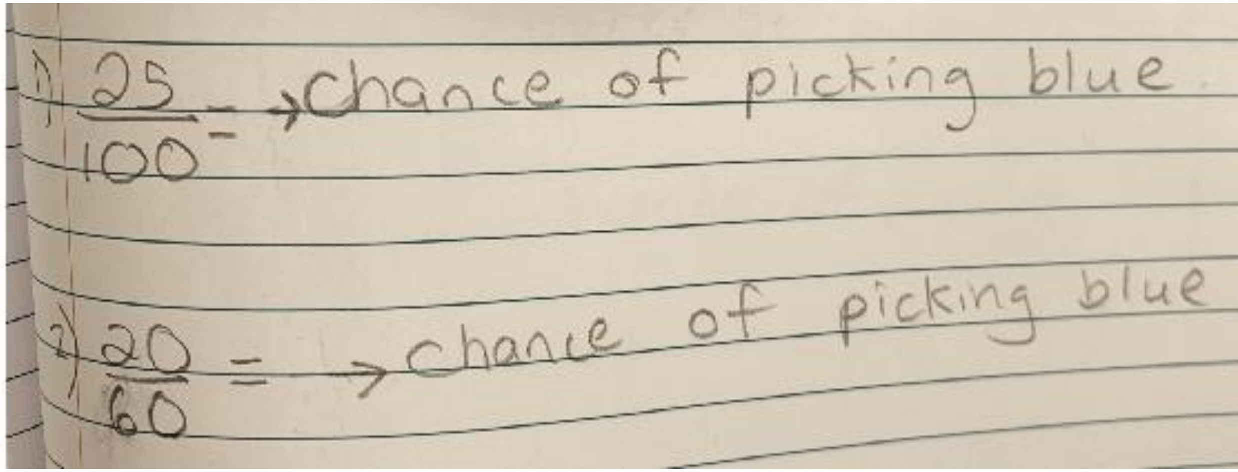
Explain your answer.



Add 10 blue marbles and remove 10 red ones?

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Telling Response – removes or lower cognitive demand

- Provides considerable information

- S: Add 10 blue marbles to Bag 2?
- T: No, that won't work, adding10 blue marbles to Bag 2 will give you 30/60.

  You need to find an equivalent fraction …

  use cross multiplication: $\frac{1}{4} = \frac{x}{60}$

Directed Guidance – lowers cognitive demand

- Redirects thinking towards teacher's own thinking

- S: Add 10 blue marbles to Bag 2?
- T: If you add 10 blue marbles to Bag 2, that changes the fraction to 30/60. What is that in percent?
- S: It's 50%
- T: But you want to make it 25% like in Bag 1. Change the fraction in Bag 2 to make it 25%.

Probing Guidance – maintains cognitive demand

- Addresses student's struggle, builds on student's thinking

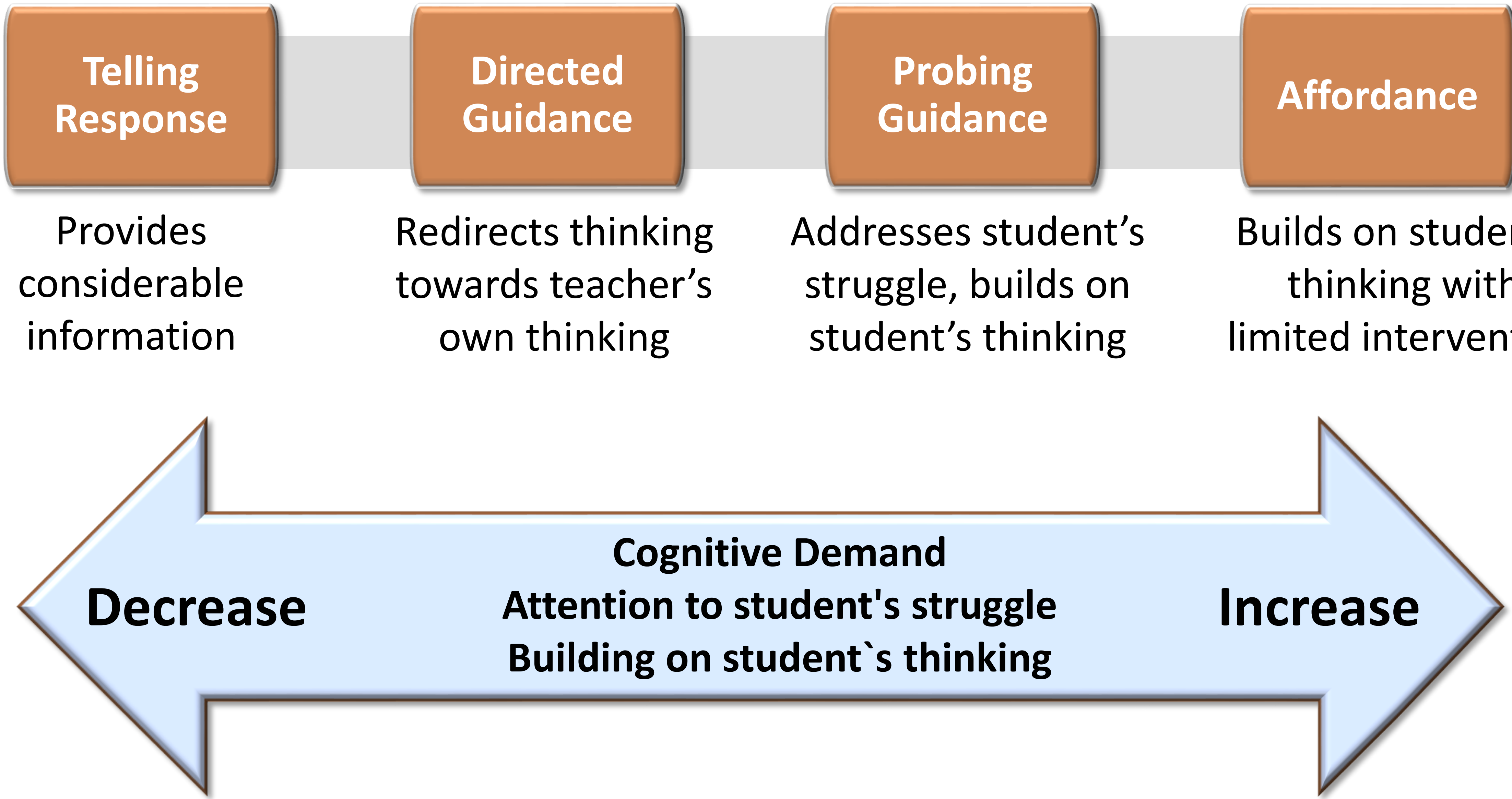
- S: Add 10 blue marbles to Bag 2?
- T: What do you mean 'add 10 marbles to Bag 2'?
- S: Make it so it's 30/60
- T: Bag 1 has 25/100 blue marbles. Is 30/60 the same as 25/100?
- S: No, I have to make Bag 2 have less blue marbles.

Affordance – maintains or raises cognitive demand

- Builds on student's thinking with limited intervention

- S: Add 10 blue marbles to Bag 2?
- T: So why don't you test it? You want Bag 2 to have the same chance of getting a blue marble as in Bag 1
- S: This would give me 30/60
- T: Would that work? Think about it for a moment.
- S: No, I have to make it equivalent to 25/100 or 25% chance.

How teachers respond to students' struggle…



(Warshauer, 2015)

JJ was diagnosed with an LD at the age of nine and is in the seventh grade. JJ has challenges with word problems presented in a verbal format. During math class, JJ walks around the room as a way to alleviate anxiety. When JJ is engaged in a math problem, he usually blurts out several responses. When asked to explain his reasoning, he sometimes feels frustrated and throws his paper on the floor.

Problem:

Sara needs a liter of paint for a project. She has $\frac{1}{5}$ of a liter and her brother has $\frac{2}{5}$ of a liter. How many liters do they have altogether?

JJ's response:

I think $\frac{1}{5} + \frac{2}{5} = \frac{3}{10}$

so they have $\frac{3}{10}$ of a liter.

JJ's response:

I think $\frac{1}{5} + \frac{2}{5} = \frac{3}{10}$ so they have $\frac{3}{10}$ of a liter.

Kind of struggles
1. Getting started
2. Carrying out a process
3. Uncertainty in explaining work
4. Express misconception and errors

Teacher A

So how did you come up with that answer ? Would you mind drawing your work ?

<returns later…>

Tell me what you think the bottom number means.

Does it make sense to have $\frac{3}{10}$ as the quantity ? Why ?

TEACHER B

But when you are adding fractions and the denominators are the same, then you keep the denominator and it does not change.

So change your answer to follow that rule.

TEACHER C

I see how you would think that it is 3/10.

You are good at adding. Let me draw a picture to show that the fifth doesn’t change.

See how we only add the numerator, not the denominator?

TEACHER D

Can you draw $\frac{1}{5}$ of a liter and $\frac{2}{5}$ of a liter ?

Now draw me $\frac{3}{10}$ of a liter.

Do you see how $\frac{3}{10}$ can`t be the right quantity ? Can you explain ?

- Telling Response
- Directed Guidance
- Probing Guidance
- Affordance

Teacher A

So how did you come up with that answer ? Would you mind drawing your work ?

<returns later…>

Tell me what you think the bottom number means.

Does it make sense to have $\frac{3}{10}$ as the quantity ? Why ?

Telling Response

Directed Guidance

Probing Guidance

Affordance

Teacher A

So how did you come up with that answer ? Would you mind drawing your work ?

<returns later…>

Tell me what you think the bottom number means.

Does it make sense to have $\frac{3}{10}$ as the quantity ? Why ?

- Makes student`s struggle visible
- Probes student`s thinking
- Less intervention
- Cognitive demand is maintained & raised
- Time given to work through struggle

Affordance

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TEACHER B

But when you are adding fractions and the denominators are the same, then you keep the denominator and it does not change.

So change your answer to follow that rule.

- Telling Response
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TEACHER B

But when you are adding fractions and the denominators are the same, then you keep the denominator and it does not change.

So change your answer to follow that rule.

- Alters the problem into a procedure
- Structured by the teacher
- Moves student toward the answer
- Student`s work is reduced to numerical manipulation
- Cognitive demand is removed

Telling

TEACHER C

I see how you would think that it is 3/10.

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Telling Response

Directed Guidance

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Affordance

Directed Guidance

TEACHER C

I see how you would think that it is 3/10.

You are good at adding. Let me draw a picture to show that the fifth doesn’t change.

You see how we only add the numerator, not the denominator?

- Redirects student thinking toward teacher`s thinking
- Built on teacher`s ideas
- Moves student toward the answer
- Deflects student`s struggle
- Cognitive demand is lowered

TEACHER D

Can you draw $\frac{1}{5}$ of a liter and $\frac{2}{5}$ of a liter ?

Now draw me $\frac{3}{10}$ of a liter.

Do you see how $\frac{3}{10}$ can`t be the right quantity ? Can you explain ?

Telling Response

Directed Guidance

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Probing Guidance

Teacher D

<u>Can you draw</u> $\frac{1}{5}$ of a liter and $\frac{2}{5}$ of a liter ?

<u>Now draw me</u> $\frac{3}{10}$ of a liter.

Do you see how $\frac{3}{10}$ can`t be the right quantity ? <u>Can you explain ?</u>

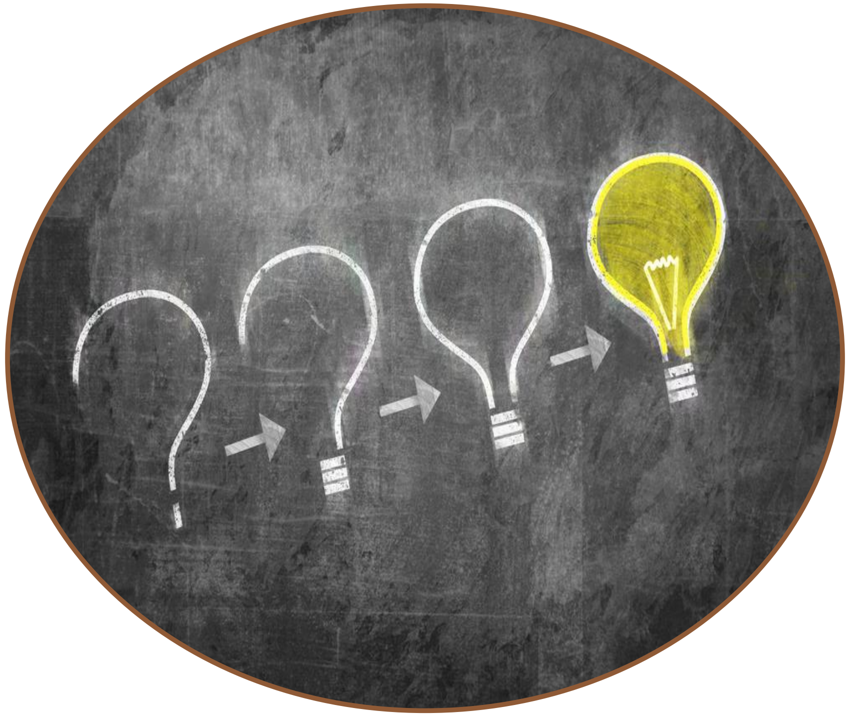
- Makes student`s error visible
- Probes student`s thinking
- Remains focused on the struggle
- Asks student for explanation
- Cognitive demand is maintained

- Balance between sustaining student engagement and maintaining cognitive demand
- Varying degrees of hints, corrections, suggestions
- Know the needs of your students with LDs

KEEP THE INTELLECTUAL WORK WITH STUDENTS

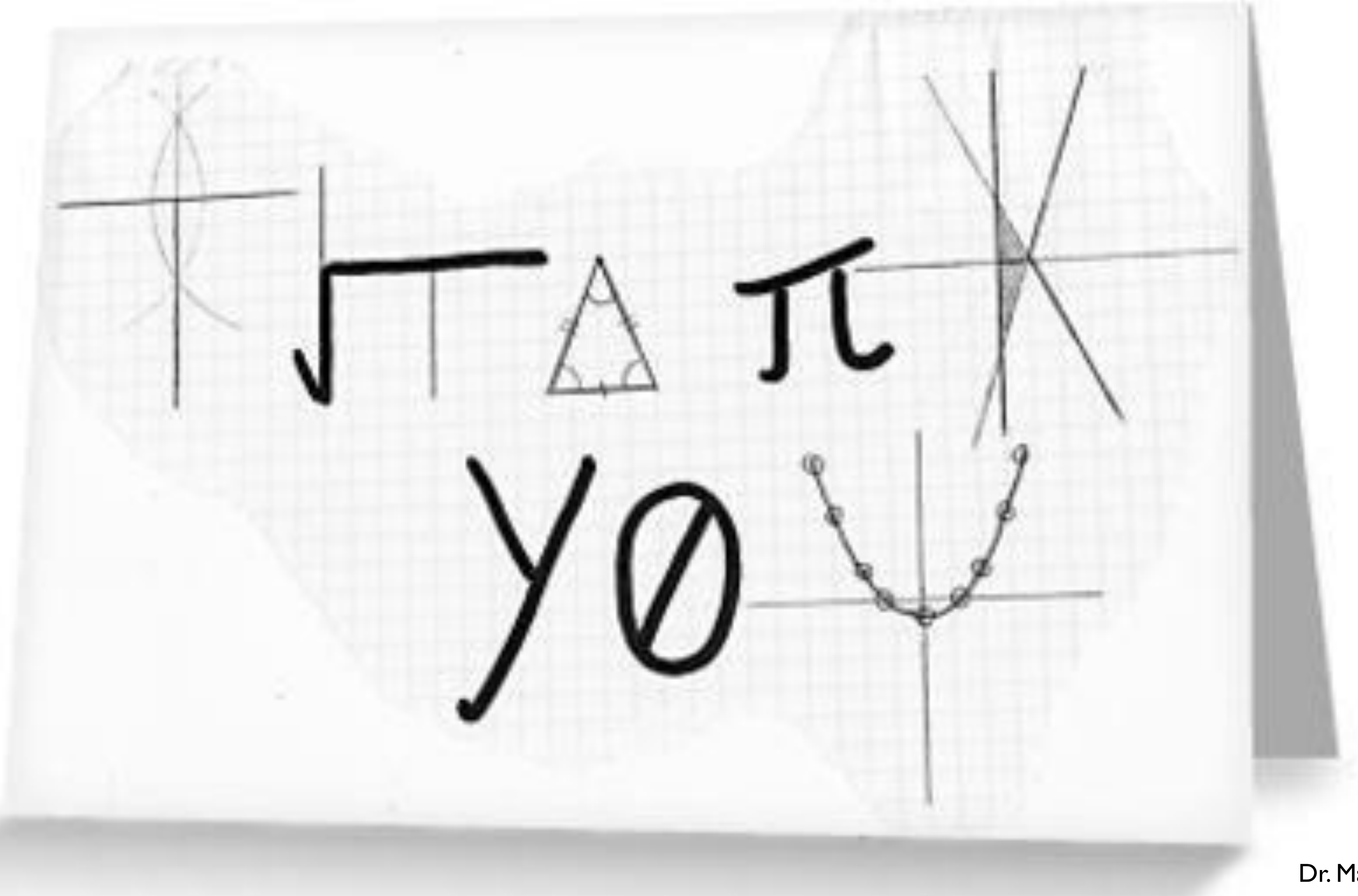
IMPLICATIONS

- Must cultivate socio-mathematical norms where struggle is embraced and understood as part of the learning culture
- Struggles in math are not impediments to learning, but rather opportunities to deepen learning
- Teacher responses should aim to:
  - identify source of struggle,
  - build on student`s ideas, probe ideas
  - give time to work through the struggle,
  - maintain and raise cognitive demand



REFERENCES

- Fuchs, L. S., Fuchs, D. (2002). Mathematical problem-solving profiles of students with mathematics difficulties with and without comorbid reading disabilities. Journal of Learning Disabilities, 35, 563–573. doi:10.1177/00222194020350060701
- Gagnon, J. C., & Maccini, P. (2001). Preparing students with disabilities for algebra. *Teaching Exceptional Children*, *34*(1), 8-15.
- Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371-404). Charlotte, NC: Information Age.
- Hiebert, J., & Wearne, D. (2003). Developing understanding through problem solving. In H. L. Schoen & R.I. Charles (Eds.), Teaching mathematics through problem solving: Grades (pp. 6–12). Reston, VA: NCTM.
- Jackson, R. R., & Lambert, C. (2010). How to Support Struggling Students. Mastering the Principles of Great Teaching series. ASCD. 1703 North Beauregard Street, Alexandria, VA 22311-1714.
- Lerner, J. (2000). Learning disabilities: Theories, diagnosis, and teaching strategies (8th ed.). Boston, MA: Houghton Mifflin.
- Lynch, S. D., Hunt, J. H., & Lewis, K. E. (2018). Productive struggle for all: Differentiated instruction. *Mathematics Teaching in the Middle School*, *23*(4), 194-201.
- Maccini, P., & Gagnon, J. C. (2000). Best practices for teaching mathematics to secondary students with special needs. *Focus on exceptional children*, *32*(5).
- Maccini, P., Hughes, C. A. (2000). Effects of a problem-solving strategy on the introductory algebra performance of secondary students with learning disabilities. Learning Disabilities Research & Practice, 15, 10–21. doi:10.1207/SLDRP1501_2
- Reid, S., & Reid, M. (2020). Dynamic Communities of Math Learners: Fostering Well-Being and Reducing Anxiety. LD@School. Retrieved from: https://www.ldatschool.ca/dynamic-communities-math-anxiety/
- Silver, E. A., & Stein, M. K. (1996). The QUASAR Project: The 'revolution of the possible' in mathematics instructional reform in urban middle schools. Urban Education, 30, 476–521.
- Warshauer, H.K., (2015a). Productive struggle in middle school mathematics classrooms. *Journal of Mathematics Teacher Education, 18*(4), 375-400.
- Warshauer, H. K. (2015b). Strategies to support productive struggle. *Mathematics Teaching in the Middle School*, *20*(7), 390-393.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